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**Nucleon mean free path in nuclear matter based on nuclear Schwinger-Dyson formalism**

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**abstract** A mean free path of nucleon moving through nuclear matter with kinetic energy of more than 100 MeV is formulated based on the bare vertex nuclear Schwinger-Dyson (BNSD) method in the Walecka model. The self-energy which is derived from the higher order diagrams more than the forth order includes the Feynman part of propagator of energetic nucleon and grows up rapidly as an increase of kinetic energy. To avoid too large growth of these diagrams, meson propagators are modified by introducing some form factors to take account of a internal structure of hadron. It is confirmed that the mean free path calculated by the BNSD method agrees good with experimental data if a reasonable form factor is chosen, i.e., a dipole (quadrupole) type of form factor with a cut-off parameter about 750 MeV  $\sim$  1000 MeV (1200 MeV  $\sim$  1500 MeV).

## 1. INTRODUCTION

The nucleon mean free path in nuclear medium is one of the important quantity to show the property of nuclear matter as well as the binding energy and the incompressibility etc. It is experimentally extracted from the proton (or neutron)-nucleus total cross sections for many targets at a spread of energy[1][2].

Recently there have been the numerical results of the nucleon mean free path in nuclear matter in the framework of relativistic approach based on the Dirac phenomenology [3] , in the relativistic impulse approximation [4][5] and in the density dependent Hartree-Fock approximation [6]. In this paper we evaluate the nucleon mean free path in nuclear matter in the  $\sigma$ - $\omega$  model based on the nuclear Schwinger-Dyson formalism .

In the last several years ,the two loop correction and the ring-sum correction were calculated in the framework of renormalizable  $\sigma - \omega$  model and these contributions to the energy density are much in the Feynman part and in the finite density part [7][8][9][10]. It is known that the large contributions of the Feynman part are reduced by introducing form factors at all vertex in the loop diagram [11][12] or by vertex corrections[13]. These procedures for reducing short-range distance contributions are qualitatively acceptable at present to take account of the internal structure of hadron. By using the bare vertex nuclear Schwinger-Dyson (BNSD) method we showed that the loop corrections in the density part were also hardly reduced because the these corrections were strongly canceled among the contributions of three components ,i.e.,  $\sigma$ - and  $\omega$ -mesons and  $\sigma$ - $\omega$  mixture [14][15].

Until now we have studied about the saturation property of ground state of nuclear matter based on the BNSD method[14][15]. In these studies, we determined two meson-nucleon coupling constants to get the minimum energy at the normal density, taking care to remove the instability near the normal density by using some recipes [16][17]. After this work, we studied optical potentials for nucleon near Fermi surface under the BNSD approximation by using the results of nuclear matter[17], and we obtained the reasonable optical potential both in the real and imaginary parts compared with empirical findings [18][19].

In this paper we estimate the mean free path of nucleon which travels through nuclear matter having the kinetic energy of more than 100 MeV above the Fermi surface. We will show that an optical potential for such a high energy nucleon outside the Fermi sphere is derived from the Feynman part of self-energy which corresponds to the higher order diagrams more than the fourth-order, besides the density part of self-energy. These higher order diagrams yield very large contributions to the optical potential both in the real and imaginary parts as well as large contributions of the Feynman part of energy density of nuclear matter. We point out that there need meson propagators modified by introducing some form factors to make the mean free path calculated by the BNSD method agree

with ones extracted in this kinetic energy region.

The organization of this paper is as follows. We develop the BNSD method for a energetic nucleon traveling far above the Fermi surface in Sec. II. The numerical results and discussion are shown in Sec. III. We summarize our work of this paper in Sec.IV.

## 2. FORMULATION

We adopt the Walecka model[20] which consists of three fields, the nucleon  $\psi$ , the scalar  $\sigma$ -meson  $\phi$  and the vector  $\omega$ -meson  $V_\mu$ . The lagrangian density is given by

$$L = -\bar{\psi}(\gamma_\mu \partial_\mu + M)\psi - \frac{1}{2}(\partial_\mu \phi \partial_\mu \phi + m_s^2 \phi^2) - (\frac{1}{4}F_{\mu\nu}F_{\mu\nu} + \frac{1}{2}m_v^2 V_\mu V_\mu) + g_s \bar{\psi}\psi\phi + ig_v \bar{\psi}\gamma_\mu \psi V_\mu, \quad (1)$$

where  $F_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu$  and  $M, m_s, m_v, g_s$  and  $g_v$  are nucleon mass,  $\sigma$ -meson mass,  $\omega$ -meson mass,  $\sigma$ -nucleon and  $\omega$ -nucleon coupling-constants respectively.

The nucleon propagator is obtained by following form,

$$G(k) = G_F(k) + G_D(k)$$

$$= \frac{-1}{i\gamma_\mu k_\mu^* + M_k^* - i\epsilon} + (-i\gamma_\mu k_\mu^* + M_k^*) \frac{i\pi}{E_k^*} \delta(k_0^* - E_k^*) \theta(k_F - |\mathbf{k}|), \quad (2)$$

$$E_k^* = \sqrt{\mathbf{k}^{*2} + M_k^{*2}}, \quad (3)$$

$$M_k^* = M + \Sigma_s(k), \quad (4)$$

$$k_\mu^* = (\mathbf{k}^*, ik_0^*)$$

$$= \left( \mathbf{k}(1 + \Sigma_v(k)), i(k_0 + \Sigma_0(k)) \right), \quad (5)$$

where  $G_F$  and  $G_D$  are the Feynman part and the density-dependent part, respectively.

The nucleon self-energy  $\Sigma(k)$  is classified as shown in Fig.1

Fig.1

$$\Sigma(k) = \Sigma_H(k) + \Sigma_{SDD}(k) + \Sigma_{SDF}(k), \quad (6)$$

where  $\Sigma_H$ ,  $\Sigma_{SDD}$  and  $\Sigma_{SDF}$  are the Hartree term, the SD density term and the SD Feynman term respectively. And this nucleon self-energy is alternatively classified according to the property of Lorentz transformation,

$$\Sigma = \Sigma_s(k) - \gamma_0 \Sigma_0(k) + i\gamma_i \cdot k_i \Sigma_v(k), \quad (7)$$

where  $\Sigma_s$ ,  $\Sigma_0$  and  $\Sigma_v$  mean the scalar component, the timelike component of vector type and the spacelike component of vector type.

In the BNSD method, the meson propagator satisfies the following Dyson equation as shown in Fig.1,

$$D(q) = D_0(q) + D_0(q)\Pi(q)D(q), \quad (8)$$

where  $D_0(q)$  denotes the free meson propagator and  $\Pi(q)$  denotes the meson self-energy. Then the meson propagator has the real and the imaginary parts,

$$D(q) = ReD(q) + iImD(q). \quad (9)$$

To derive the imaginary part of self-energy we execute a Wick rotation  $q_0 \rightarrow iq_0$  in the  $q_0$ -integral of the SD Feynman term and rewrite  $\Sigma_{SDF}$  as follows,

$$\Sigma_{SDF} = \Sigma_{SDF}^{on} + \Sigma_{SDF}^{off}, \quad (10)$$

$$Re\Sigma_{SDF} = Re\Sigma_{SDF}^{on} + Re\Sigma_{SDF}^{off}, \quad (11)$$

$$Im\Sigma_{SDF} = Im\Sigma_{SDF}^{on}. \quad (12)$$

The superscripts "on" or "off" of  $\Sigma_{SDF}$  denote that the nucleon propagator included in the SD diagram is on-shell or off-shell, respectively. We note that the real part of  $\Sigma_{SDF}$  is composed of two terms, i.e., a finite term  $Re\Sigma_{SDF}^{on}$  and an infinite term  $Re\Sigma_{SDF}^{off}$ , while the imaginary part of  $\Sigma_{SDF}$  is only one term, a finite  $Im\Sigma_{SDF}^{on}$ . We drop the infinite real self-energy as stated in the previous paper [17].

We obtain the explicit expression of real and imaginary nucleon self-energies corresponding to the Feynman diagrams given in Fig.2 as follows,

Fig.2

$$\begin{aligned} Re\Sigma_s(k) = & -\frac{2}{\pi^2} \frac{g_s^2}{m_s^2} \rho_s - \frac{g_s^2}{8\pi^2} \int_0^{k_F} q^2 \frac{M_q^*}{E_q^*} dq \int_{-1}^1 dx \Delta_0(R) \\ & + \frac{g_v^2}{8\pi^2} \int_0^{k_F} q^2 \frac{M_q^*}{E_q^*} dq \int_{-1}^1 dx 4D_0(R) \\ & + \frac{g_s^2}{8\pi^2} \int_{k_F}^k q^2 dq \int_{-1}^1 dx \frac{M_q^*}{E_q^*} \left[ \frac{R^2}{\vec{R}^2} (\Delta_0 \Pi_m D_m)_R + (\Delta_0 \Pi_s D_s)_R \right] \end{aligned}$$

$$\begin{aligned}
& + \frac{g_v^2}{8\pi^2} \int_{k_F}^k q^2 dq \int_{-1}^1 dx \frac{M_q^*}{E_q^*} [-4(D_0 \Pi_t D_t)_R + \frac{R^2}{\bar{R}^2} \{-(D_0 \Pi_l D_l)_R + (D_0 \Pi_t D_t)_R - \frac{R^2}{\bar{R}^2} (D_0 \Pi_m D_m)_R\}] \\
& + \frac{2g_s g_v}{8\pi^2} \int_{k_F}^k q^2 \int_{-1}^1 dx \frac{R^2}{\bar{R}^2} \text{Re} D_m(R), \tag{13a}
\end{aligned}$$

$$\text{Im} \Sigma_s(k) = \frac{g_s^2}{8\pi^2} \int_{k_F}^k q^2 \frac{M_q^*}{E_q^*} dq \int_{-1}^1 dx \text{Im} D_s(R)$$

$$\begin{aligned}
& - \frac{g_v^2}{8\pi^2} \int_{k_F}^k q^2 \frac{M_q^*}{E_q^*} dq \int_{-1}^1 dx \{4\text{Im} D_l(R) + (3 + \frac{R_0^2}{\bar{R}^2})(\text{Im} D_t(R) - \text{Im} D_l(R))\} \\
& + \frac{g_s g_v}{4\pi^2} \int_{k_F}^k q^2 dq \int_{-1}^1 dx \text{Im} D_m(R) \frac{R^2}{\bar{R}^2}, \tag{13b}
\end{aligned}$$

$$\begin{aligned}
\text{Re} \Sigma_0(k) &= -\frac{2}{\pi^2} \frac{g_v^2}{m_v^2} \frac{k_F^3}{3} + \frac{g_s^2}{8\pi^2} \int_0^{k_F} q^2 dq \int_{-1}^1 dx \Delta_0(R) \\
& + \frac{g_v^2}{8\pi^2} \int_0^{k_F} q^2 dq \int_{-1}^1 dx 2D_0(R) \\
& - \frac{g_s^2}{8\pi^2} \int_{k_F}^k q^2 dq \int_{-1}^1 dx [\frac{R^2}{\bar{R}^2} (\Delta_0 \Pi_m D_m)_R + (\Delta_0 \Pi_s D_s)_R]
\end{aligned}$$

$$\begin{aligned}
& + \frac{g_v^2}{8\pi^2} \int_{k_F}^k q^2 dq \int_{-1}^1 dx [-2(D_0 \Pi_t D_t)_R - \frac{R^2}{\bar{R}^2} \{-(D_0 \Pi_l D_l)_R + (D_0 \Pi_t D_t)_R - \frac{R^2}{\bar{R}^2} (D_0 \Pi_m D_m)_R\}] \\
& - \frac{2g_s g_v}{8\pi^2} \int_{k_F}^k q^2 \int_{-1}^1 dx \frac{M_q^*}{E_q^*} \frac{R^2}{\bar{R}^2} \text{Re} D_m(R), \tag{14a}
\end{aligned}$$

$$\text{Im} \Sigma_0(k) = -\frac{g_s^2}{8\pi^2} \int_{k_F}^k q^2 dq \int_{-1}^1 dx \text{Im} D_s(R)$$

$$\begin{aligned}
& - \frac{g_v^2}{8\pi^2} \int_{k_F}^k q^2 dq \int_{-1}^1 dx \{2\text{Im} D_l(R) + (2 + \frac{R^2}{\bar{R}^2})(\text{Im} D_t(R) - \text{Im} D_l(R))\} \\
& - \frac{g_s g_v}{4\pi^2} \int_{k_F}^k q^2 dq \int_{-1}^1 dx \frac{M_q^*}{E_q^*} \frac{R^2}{\bar{R}^2} \text{Im} D_m(R), \tag{14b}
\end{aligned}$$

$$\begin{aligned}
Re\Sigma_v(k) = & \frac{g_s^2}{8\pi^2 k^2} \int_0^{k_F} q^2 dq \int_{-1}^1 dx \frac{q^* kx}{E_q^*} \Delta_0(R) \\
& + \frac{g_v^2}{8\pi^2 k^2} \int_0^{k_F} q^2 dq \int_{-1}^1 dx \frac{q^* kx}{E_q^*} 2D_0(R) \\
& - \frac{g_s^2}{8\pi^2 k^2} \int_{k_F}^k q^2 dq \int_{-1}^1 dx \frac{q^* kx}{E_q^*} \left[ \frac{R^2}{\vec{R}^2} (\Delta_0 \Pi_m D_m)_R + (\Delta_0 \Pi_s D_s)_R \right] \\
& + \frac{g_v^2}{8\pi^2 k^2} \int_{k_F}^k q^2 dq \int_{-1}^1 dx \frac{q^* kx}{E_q^*} \left[ -2(D_0 \Pi_t D_t)_R - \frac{R^2}{\vec{R}^2} \{ -(D_0 \Pi_l D_l)_R + (D_0 \Pi_t D_t)_R - \frac{R^2}{\vec{R}^2} (D_0 \Pi_m D_m)_R \} \right]
\end{aligned} \tag{15a}$$

$$\begin{aligned}
Im\Sigma_v(k) = & -\frac{g_s^2}{8\pi^2 k^2} \int_{k_F}^k q^2 dq \int_{-1}^1 dx \frac{q^* kx}{E_q^*} ImD_s(R) \\
& - \frac{g_v^2}{8\pi^2 k^2} \int_{k_F}^k q^2 dq \int_{-1}^1 dx \frac{q^* kx}{E_q^*} \{ 2ImD_l(R) + (2 - \frac{R^2}{\vec{R}^2})(ImD_t(R) - ImD_l(R)) \},
\end{aligned} \tag{15b}$$

where  $R = k - q$ , and  $\rho_s$  and  $\rho_B$  denote the scalar and the baryon densities respectively, and  $\Delta_0$  and  $D_0$  denote the free propagators of  $\sigma$ - and  $\omega$ -mesons, respectively. The subscripts s,l,t and m of  $D(R)$  and  $\Pi(R)$  denote the component of  $\sigma$ -meson, the longitudinal and transverse components of  $\omega$ -meson and the component of mixture of  $\sigma$ - and  $\omega$ -mesons, respectively. The detailed expressions of  $D(R)$  are given in Ref.[16] and the analytical expressions of  $\Pi(R)$  are given in Ref.[10].

The last three terms in Eqs.(13a) and (14a) and the last two terms in Eq.(15a) are contributed from the same higher order diagrams which yield the imaginary part.

We modify the meson propagators in the SD Feynman part of self-energy by introducing some form factors at the vertices in agreement with the point of view in Ref. [11][12][21] to examine the short range interactions and the finite size of hadrons ,as follows

$$D(q^2) \rightarrow [F(q^2)]^2 \cdot D(q^2), \tag{16}$$

where  $F(q^2) = 1/[1 + q^2/\Lambda^2]^n$ . The case of n=1 is a monopole type of form factor and n=2 is a dipole type, etc.

Optical potentials are defined from self-energies  $\Sigma$  as

$$U_S = \frac{\Sigma_s - M\Sigma_v}{1 + \Sigma_v} = U_{SR} + iU_{SI}, \tag{17}$$

$$U_V = \frac{-\Sigma_0 + E\Sigma_v}{1 + \Sigma_v} = U_{VR} + iU_{VI}, \quad (18)$$

where  $E$  is the energy of objective nucleon propagating with the momentum  $k$  satisfying the following dispersion relation,

$$E = \sqrt{\mathbf{k}^2 + (M + U_S)^2} + U_V. \quad (19)$$

Eq.(19) is rewritten as

$$\frac{\mathbf{k}^2}{2M} + V + iW = E - M + \frac{(E - M)^2}{2M}, \quad (20)$$

with Schrödinger equivalent potential form

$$V = U_{SR} + U_{VR} + \frac{(E - M)}{M}U_{VR} + \frac{1}{2M}(U_{SR}^2 + U_{VI}^2 - U_{SI}^2 - U_{VR}^2), \quad (21)$$

$$W = U_{SI} + U_{VI} + \frac{(E - M)}{M}U_{VI} + \frac{1}{M}(U_{SR}U_{SI} - U_{VR}U_{VI}). \quad (22)$$

The nucleon momentum is complex as well as the optical potentials and can be expressed as

$$|\mathbf{k}| = k_R + ik_I. \quad (23)$$

and then we obtain the nucleon mean free path as follows,

$$\lambda = \frac{1}{2k_I} = \frac{1}{2} \left\{ -M \left( E - M - V + \frac{(E - M)^2}{2M} \right) + M \left[ \left( E - M - V + \frac{(E - M)^2}{2M} \right)^2 + W^2 \right]^{1/2} \right\}^{-1/2}. \quad (24)$$

### 3. RESULTS AND DISCUSSION

In this section we calculate self-energies of the nucleon propagating with the energy  $E$  and the momentum  $\mathbf{k}$ , convert them into the optical potentials in the Schrödinger equivalent form, and evaluate the nucleon mean free path from Eq.(24). The relation between the energy  $E$  and the momentum  $k$  is given by approximating Eq.(19) as follows,

$$E = \sqrt{\mathbf{k}^2 + (M + \text{Re}\Sigma_s(E, k))^2} - \text{Re}\Sigma_0(E, k). \quad (25)$$

The nucleon momentum  $\mathbf{k}$ , the upper limit of integral in the real and imaginary self-energy, is determined from Eq.(25) by putting the nucleon energy  $E = E_{in} + M$ , where  $E_{in}$  is the nucleon incident energy.

In this paper we determine the coupling constants of the  $\sigma$ -nucleon and  $\omega$ -nucleon to satisfy the saturation property of nuclear matter at the normal density  $\rho=0.170 \text{ fm}^{-3}$  ( $k_F = 1.36 \text{ fm}^{-1}$ ), and  $0.193 \text{ fm}^{-3}$  ( $k_F = 1.42 \text{ fm}^{-1}$ ) and summarize the parameters in Table.

Table

Using these parameters (we choose the parameters in the case of the normal density  $\rho=0.17 \text{ fm}^{-3}$ ), we calculate the nucleon mean free path. We start with evaluating the higher order diagrams more than the fourth order in the nucleon self-energy shown in Fig.2. Since these diagrams include the Feynman propagator of nucleon on-shell in the intermediate state, we are afraid whether these diagrams give a large contribution to the self-energy. In Fig.3, we show contributions of  $\sigma$ -meson,  $\omega$ -meson and  $\sigma$ - $\omega$  mixture to  $\text{Re}\Sigma_s$  and  $\text{Re}\Sigma_0$ , respectively. As the increase of kinetic energy of nucleon,  $E - M$ , each component,  $\Sigma^\sigma$ ,  $\Sigma^\omega$  and  $\Sigma^{\sigma-\omega}$ , grows almost linearly, because the meson propagator  $\sim 1/q^2$  and the Jacobian  $q^2$  factor cancel each other in  $q$ -integrals of self-energies with the upper limit of  $k$ . In  $\text{Re}\Sigma_s$ , additive contributions of  $\sigma$ -meson and  $\sigma$ - $\omega$  mixture cancel strongly with the one of  $\omega$ -meson and as a result the net contribution is small. On the other hand, contributions of three components to  $\text{Re}\Sigma_0$  are all additive and as a result the self-energy is very huge and the net contribution becomes nearly equal with the Hartree contribution.

Fig.3(a), Fig.3(b)

In Fig.4, we also show the contributions of three components to  $\text{Im}\Sigma_s$  and  $\text{Im}\Sigma_0$ , respectively. The contributions of  $\sigma$ -meson and  $\sigma$ - $\omega$  mixture to  $\text{Im}\Sigma_0$  are far smaller than the one of  $\omega$ -meson. The  $\omega$ -meson dominance should be marked. The value of  $\text{Im}\Sigma_0$  at  $E - M = 200 \text{ MeV}$  are considerably reduced in comparison with the one derived from the fourth order diagrams in Ref.[21]. The higher order contribution more than the fourth order is very important [17]. The value of  $\text{Im}\Sigma_s$  is also reduced at the same kinetic energy and, however, has the negative sign when  $E - M > 350 \text{ MeV}$ , which is opposite to the sign of  $\text{Im}\Sigma_s$  obtained from the fourth order diagram. The change of sign is easily understood from Fig.4(b). In lower kinetic energy region  $\text{Im}\Sigma_s$  is positive because the contribution of  $\omega$  meson is large and positive compared with the small and negative contributions of others. On the other hand, in higher kinetic energy region, the sign of  $\text{Im}\Sigma_s$  changes to the negative one because the contribution of  $\omega$  meson changes to the negative one and becomes large. It should be remarked that  $\text{Im}\Sigma$  originates from the Feynman part of self-energy. So, although  $\text{Im}\Sigma$  is small if  $E - M < 100 \text{ MeV}$ , it grows up unphysically if  $E - M > 100 \text{ MeV}$  as well as the vacuum effect for the energy density in nuclear matter is unphysically large. Then, we introduce form factors into each vertex by modifying meson propagators,  $D(q) \rightarrow [f(q^2)]^2 \cdot D(q)$  as shown in Ref.[21]. It is noted that, since the SD diagrams are composed of full meson propagators with a ring-sum



correction, a dipole (quadrupole) type of form factor introduced into the SD diagram corresponds to a monopole (dipole) type of form factor into the fourth order diagram.

Fig.4(a), Fig.4(b)

There are two kinds of  $\Sigma_{SDF}$  leading to the real self-energy. One is the above-discussed self-energy derived from the same diagram which yields the imaginary part. The other is the self-energy derived from the diagrams including the Feynman propagator of nucleon off-shell. The renormalization procedure showed that  $\text{Re}\Sigma_{SDF}$  is very large. So we must develop a new recipe which takes account of the size of hadron. Then, we should evaluate the two diagrams at the same time when we want to know their contributions to the real self-energy. In the present situation, therefore, we do not pick up both of them though we are afraid of the violation of the dispersion relation between the real part and the imaginary part.

In Fig.5(a) and (b), we show the imaginary part and the real part of Schrödinger equivalent potentials, in cases of two types of form factor, the dipole with  $\Lambda = 750$  (solid curve), 1000 (dotted curve) and 1500 MeV (dashed curve) and the quadrupole with  $\Lambda = 1200$  (bold solid curve) and 1500 MeV (bold dotted curve). The solid dots are empirical information [18][19]. As the kinetic energy increases, the imaginary potential decreases slowly if  $E - M < 300\text{MeV}$  and rapidly if  $E - M > 350\text{MeV}$ . This feature is particularly in character with the dashed curve which is closer to the one without the form factor. The reason is as follows. In the rough, the imaginary potential is proportional to  $\text{Im}\Sigma_s - \text{Im}\Sigma_0$  and  $\text{Im}\Sigma_s$  changes from positive sign to negative sign at  $E - M \simeq 350\text{MeV}$ . The numerical data of the dipole type of form factor with  $\Lambda = 750$  MeV almost correspond to the data of the quadrupole type with  $\Lambda = 1200$  MeV. Similarly, the dipole type with  $\Lambda = 1000$  MeV corresponds to the quadrupole type with  $\Lambda = 1500$  MeV. Our numerical results are a little different from empirical information in the tendency in the low kinetic energy region.

Fig.5(a), Fig.5(b)

The nucleon mean free path is written by using Schrödinger equivalent potential form as given by Eq.(24). In Fig.6 we show the mean free path in the cases of some form factors. When we choose the dipole type of form factor with cut off  $\Lambda=750$  MeV and the quadrupole type of one with  $\Lambda=1200$  MeV, the values of mean free path are good agreement with the experimental ones, but the experimental data are extracted from the reaction of proton and nuclei and so we consider that the reasonable form factor is the dipole type with  $\Lambda \simeq 750 \sim 1000$  MeV or the quadrupole type with  $\Lambda \simeq 1200 \sim 1500$  MeV for nuclear matter.

Fig.6

Furthermore, we calculated the similar calculations using coupling constants determined at the normal density  $\rho=0.193$  ( $k_F=1.42\text{ fm}^{-1}$ ), but the results were nearly equal.

#### 4. SUMMARY

We evaluated the nucleon mean free path of the energetic nucleon based on the BNSD method and compare it with the ones extracted experimentally [1], and took a good agreement in the case of modification of meson propagators by introducing dipole type of form factor  $\Lambda = 750 \text{ MeV} \sim 1000 \text{ MeV}$  and quadrupole type with  $\Lambda = 1200 \text{ MeV} \sim 1500 \text{ MeV}$ . We confirmed that the BNSD method was useful for the derivation of optical potential for a nucleon traveling far above the Fermi surface if we consider the finite size of nucleon or some effects of the short-range corrections of nuclear force.

The real self-energy  $\text{Re}\Sigma_{SDF}^{on}$  grows unphysically as the increase of kinetic energy. Even if it can be reasonably reduced by modifying meson propagators, the residual Feynman self-energy  $\text{Re}\Sigma_{SDF}^{off}$  is expected also very large. We assure again the conclusion discussed in the previous paper [17] that we do not pick up  $\text{Re}\Sigma_{SDF}^{on}$  when we drop out  $\text{Re}\Sigma_{SDF}^{off}$ . On the other hand the imaginary part of  $\Sigma_{SDF}$  is only  $\text{Im}\Sigma_{SDF}^{on}$ . The cut off parameter of form factor introduced into  $\text{Im}\Sigma_{SDF}^{on}$  looks somewhat small but reasonable when we regard this parameter as a parameter of nucleon size.

The real part of optical potential in the Schrödinger equivalent form increases linearly as the increase of kinetic energy, keeping a good agreement with the experimental findings in the lower kinetic energy.

There remains a problem in the imaginary part of optical potential that its magnitude is too small in the lower kinetic energy region when a form factor is chosen to obtain the experimental data of mean free path in the higher kinetic energy region. In our previous work and this work, we took account of the vacuum part of meson self-energy with the low 4-momentum transfer to remove a instability around the normal density. The vacuum part of meson self-energy is also inserted into the denominator of the meson propagator in the integral of  $\text{Im}\Sigma_{SDF}^{on}$ . The vacuum part in this case has the high 4-momentum transfer and so is expected too much. As our next work, we will study the role of the vacuum effect for meson self-energy in the imaginary potential.

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### Table and Figure captions

Table Baryon density (in  $fm^{-3}$ ), coupling constants,  $\sigma$  and  $\omega$  meson masses (in MeV) in our calculation.

Fig.1 Feynman diagrams for the calculation of scattering problem based on BNSD method. Double solid (dotted) lines represent exact nucleon(meson) propagators, and single solid(dotted) lines represent free ones.

Fig.2 Feynman diagrams for the calculation of the self-energy. (a)Hartree diagram, (b)Fock diagram, (c) Higher order diagram more than fourth order. Doubly downward lines denote the hole states with momenta under the Fermi momentum  $k_F$  and the doubly upward lines denote the intermediate states with momenta  $\leq k$ .

Fig.3 The real part of nucleon self-energy extracted from Feynman propagator of nucleon on-shell. (a) The component of scalar part and (b) the component of vector part.

Fig.4 The imaginary part of nucleon self-energy extracted from Feynman propagator of nucleon on-shell. (a) The component of vector part and (b) the component of scalar part.

Fig.5 Optical potential in the Schrödinger equivalent form. (a) The imaginary part and (b) the real part.

Fig.6 The nucleon mean free path in nuclear matter. The solid dots with error bars are experimental results from Ref.[1].

### Table

$\rho_B$	$m_s$	$m_v$	$g_s$	$g_v$
0.170	550	783	9.59	11.67
0.192	550	783	9.12	11.00